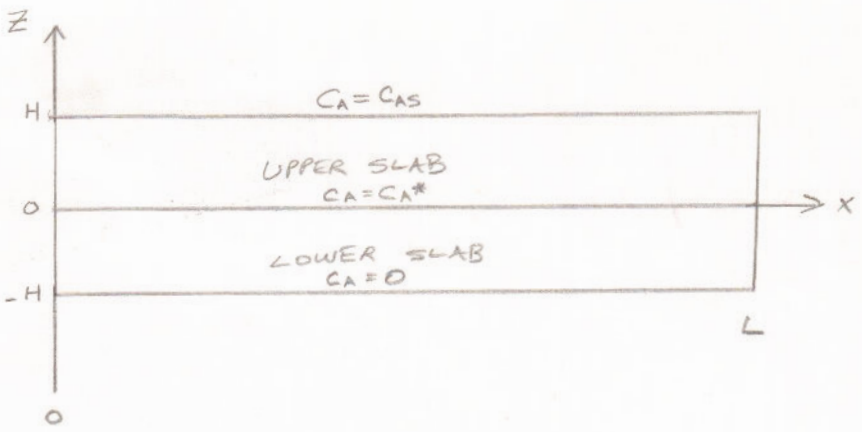
(9)



MASS BALANCE:

$$0 = S \cdot N_A\big|_{z=z} - S \cdot N_A\big|_{z=z+\Delta z} + 0$$

S = CROSS-SECTIONAL AREA

$= L^2$

$$N_A L^2\big|_{z=z} - N_A L^2\big|_{z=z+\Delta z} = 0$$

TAKING LIMIT,

$$\underline{\underline{-\frac{dN_A}{dz} = 0}} \qquad \text{FOR BOTH SLABS}$$

FICK'S LAW:

$$\vec{N}_A = -cD_{AB}\vec{\nabla}x_A + x_A\sum\vec{N}_i = -cD_{AB}\frac{dx_A}{dz} + x_A(N_A + N_B)$$

ASSUME $x_A \sim 0$

$$\vec{N}_A = -cD_{AB}\frac{dx_A}{dz} = -D_{AB}\frac{dc_A}{dz}$$

UPPER SLAB: $\underline{\underline{N_A = -D_{AU}\frac{dc_A}{dz}}}$ ---- $\frac{d}{dz}\left(D_{AU}\frac{dc_A}{dz}\right) = 0$

LOWER SLAB: $\underline{\underline{N_A = -D_{AL}\frac{dc_A}{dz}}}$ ---- $\frac{d}{dz}\left(D_{AL}\frac{dc_A}{dz}\right) = 0$

BOUNDARY CONDITIONS

	UPPER SLAB		LOWER SLAB	
①	$z = 0$	$c_A = c_A^*$	$z = 0$	$c_A = c_A^*$
②	$z = H$	$c_A = c_{AS}$	$z = -H$	$c_A = 0$

EVALUATE c_A^* BY NOTING $N_{AU} = N_{AL}$

$$D_{AU}\frac{dc_A}{dz} = D_{AL}\frac{dc_A}{dz}$$

UPPER

$$\frac{d}{dz}\left(D_{AU}\frac{dc_A}{dz}\right) = 0$$

$$\frac{d^2c_A}{dz^2} = 0$$

$$c_A = B'z + C'$$

B.C. #1: $c_A^* = C'$

B.C. #2: $c_{AS} = B'H + c_A^*$

$$B' = \frac{c_{AS} - c_A^*}{H}$$

$$c_A = \left(c_{AS} - c_A^*\right)\frac{z}{H} + c_A^*$$

$$\frac{dc_A}{dz} = \frac{c_{AS} - c_A^*}{H}$$

LOWER

$$\frac{d}{dz}\left(D_{AL}\frac{dc_A}{dz}\right) = 0$$

$$\frac{d^2c_A}{dz^2} = 0$$

$$c_A = B''z + C''$$

#1 $c_A^* = C''$

#2 $0 = -B''H + c_A^*$

$$B'' = \frac{c_A^*}{H}$$

$$c_A = c_A^*\frac{z}{H} + c_A^*$$

$$\frac{dc_A}{dz} = \frac{c_A^*}{H}$$

$N_{AU} = N_{AL}$

$$D_{AU}\left(\frac{c_{AS} - c_A^*}{H}\right) = D_{AL}\frac{c_A^*}{H}$$

$$D_{AU}c_{AS} - D_{AU}c_A^* = D_{AL}c_A^*$$

a)

$$\boxed{c_A^* = \frac{D_{AU}c_{AS}}{D_{AL} + D_{AU}}}$$

b) FLUX $\Rightarrow$ $N_A = -D_{AL}\frac{dc_{A(L)}}{dz} = -D_{AL}\frac{c_A^*}{H} = \boxed{-\left(\frac{D_{AL}D_{AU}}{D_{AL} + D_{AU}}\right)\left(\frac{c_{AS}}{H}\right)}$

NOTE: FLUX IS IN NEGATIVE z-DIRECTION, FROM HIGH TO LOW A CONCENTRATION

c) $D_{AU} = 2D_{AL}$

$$c_A^* = \frac{2D_{AL}c_{AS}}{D_{AL} + 2D_{AL}} = \frac{2}{3}c_{AS}$$